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GENERATION OF INSTABILITY WAVES AT A LEADING EDGE

invited paper

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ABSTRACT

This paper describes the generation of instability waves downstream of a leading edge by an imposed upstream disturbance. Two cases are considered. The first is concerned with mean flows of the Blasius type wherein the instabilities are represented by Tollmien-Schlichting waves. It is shown that the latter are generated fairly far downstream of the edge and are the result of a wave length reduction process that tunes the free stream disturbances to the Tollmien-Schlichting wave length. The other case is concerned with inflectional, uni-directional, transversely sheared mean flows. Such idealized flows provide a fairly good local representation to the nearly parallel flows in jets. They can support inviscid instabilities of the Kelvin-Helmholtz type. The various mathematically permissible mechanisms that can couple these instabilities to the upstream disturbances are discussed. The results are compared to some acoustic measurements and con- clusions are drawn about the generation of the instabilities in these flows.

I. GENERATION OF TOLLMIEN-SCHLICHTING WAVES IN A BOUNDARY LAYER

A. Background

It is now well established that there are many flows wherein the boundary layer turbulence is a direct result of the amplification of linear spatially growing instability waves (i.e., Tollmien-Schlichting waves) in the laminar portion of the boundary layer. These waves grow as they propagate downstream and, at least initially, the two dimensional waves exhibit the most rapid growth rates. However, once the Tollmien-Schlichting waves reach a certain amplitude, nonlinear effects rapidly set in and produce significant lateral energy transfer, which ultimately distorts the two-dimensional character of the flow. This stretches the vortex filaments and thereby produces further increases in the unsteady velocity until the flow breaks down into bursts of turbulent like motion. At this point, the boundary layer is well on its way to becoming turbulent. The length of laminar boundary layer over which these nonlinear phenomena occur is often significantly shorter than the length over which the instability waves are governed by a linear equation (namely the Orr-Sommerfeld equation). The transition point (or, more precisely, the transition Reynolds number based on the distance from the leading edge) can, therefore, be predicted from linear theory in these flows.

It is also well known (Schubauer & Skramstad[1], and Spangler & Wells[2]) that the transition Reynolds number is strongly affected by the level of turbulence in the free stream. Schubauer & Skramstad[1] showed that the transition Reynolds number of a flat plate boundary layer increases with decreasing free stream turbulence until the intensity drops below about 0.1%. At lower intensities, the transition Reynolds number remained relatively constant at 2.8×10^6. However, Spangler and Wells[3] found that they could increase the transition Reynolds number to about 5.2×10^6. They attributed this increase to the fact that background acoustic disturbances (noise) represented only a small fraction of the measured 'turbulence' level in their experiment which implies that even random acoustic disturbances may be more efficient in generating turbulence than free stream turbulence.

After it was discovered that free stream turbulence and random background acoustic disturbances can have an important effect on transition, it was natural to study the effect of a regular two dimensional small amplitude free stream oscillation of a single frequency, say ω, imposed on a two dimensional steady flow with uniform upstream velocity, say U_∞. In such a flow, the streamwise component $u_\infty(x)$ of the free stream velocity at the outer edge of the boundary layer is of the form

$$u_\infty(x) = U_0(x) + u_1(x)\, e^{-i\omega t} \qquad (1)$$

where we suppose that the unsteady streamwise velocity amplitude $u_1(x)$ is much smaller than the mean streamwise velocity U_0, x denotes the streamwise distance measured along the surface of the body and nondimensionalized by U_∞/ω, and t denotes the time. Such a flow was studied by Obremski and Fejer[3] and Miller & Fejer[4]. However, they produced the unsteady motion with a variable speed rotating shutter valve downstream of the test section of their wind tunnel and their unsteady motion could probably not be considered to be two dimensional. They measured transition Reynolds numbers and showed that they were significantly affected by the amplitude $|u_1|$ of the imposed free stream disturbance, but they did not make any measurements that would allow them to determine how the imposed disturbance affected the Tollmien-Schlichting waves. This disturbance may have generated the Tollmien-Schlichting waves directly or it may only have affected their growth rates by changing the stability characteristics of the boundary layer. It could even happen that the Tollmien-Schlichting waves were bypassed in these experiments and the free stream disturbance was able to generate turbulence directly by some nonlinear process (Morkovin[5]).

It is, therefore, important to measure the effect of the imposed disturbance on the Tollmien-Schlichting waves themselves. This was done by Shapiro[6] whose unsteady disturbance was quite two-dimensional. We will discuss his results subsequently.

We listed three mechanisms by which free stream disturbances might affect transition. However, only the first of these is truly linear in the sense that it can be described by equations which are linear in the unsteady flow perturbation. The other mechanisms would invoke terms that are quadratic in the unsteady motion. We, therefore, anticipate that the first mechanism will dominate when the amplitude of the imposed unsteadiness is sufficiently small.

B. General Linear Theory

The physics of this linear interaction will now be described.

The relevant mathematical problem has been solved numerically for a flat plate by Murdock[16] and analytically in the general case by Goldstein[12]. The following discussion is mainly based on the analysis of ref. 12.

When we say that the Tollmien-Schlichting waves are generated by the free stream disturbances, we imply that they are solutions to a well defined boundary value problem. But, the Tollmien-Schlichting waves are eigensolutions of the Orr-Sommerfeld equation (which is obtained by linearizing the Navier-Stokes equations about the mean flow and assuming that the latter is nearly parallel, which is usually a good approximation in the boundary layer). Moreover, it is well known that one can always add an arbitrary multiple of an eigensolution to the solution of a boundary value problem and still satisfy all the imposed boundary conditions. This raises the question of how Tollmien-Schlichting waves can be coupled with the imposed free stream disturbance. But, the spatially growing Tollmien-Schlichting waves will only be eigensolutions of the Orr-Sommerfeld equation when no upstream (initial) conditions are imposed (i.e., they are eigensolutions when the mean parallel flow extends from $-\infty$ to $+\infty$). The coupling comes about when upstream boundary conditions (i.e., initial conditions) are imposed.

However, the initial conditions cannot be applied directly to the solution of the Orr-Sommerfeld equation. Near the leading edge of the boundary layer (actually within a region that occupies the first few wavelengths of the boundary layer) the wave length of the disturbance is very long compared to the boundary layer thickness, and the streamwise derivatives are small. The divergence of the mean flow has a fist order effect on the unsteady motion rather than being a higher order effect that can be treated as a 'slowly varying' correction to classical parallel flow stability theory. In this region, inertia terms involving the cross stream component of the mean flow velocity have to be included in the lowest order equation for the unsteady flow. However, one can neglect unsteady pressure fluctuations across the mean boundary layer, which is still relatively thin (on a wave length scale). The flow is then governed by the linearized unsteady boundary layer equation rather than by an Orr-Sommerfeld equation with slowly varying coefficients.

This latter equation, whose eigensolutions are the Tollmien-Schlichting waves, is only valid further downstream. The upstream initial condition for the solution to this equation should, therefore, be that it 'match', preferably in the 'matched asymptotic expansion' sense, onto a solution of the unsteady boundary layer equation in some intermediate region that overlaps the unsteady boundary layer and Orr-Sommerfeld regions.

For definiteness, we restrict the discussion to flat plates whose 'nose radii' are $O(U_\infty/\omega)$. We also suppose that the characteristic wave number of u_1 is $O(\omega/U_\infty)$. The asymptotic expansion (alluded to above) is carried out in terms of the inverse Reynolds number based on the 'convective' wave length U_∞/ω of the disturbance raised to the 1/6th power, i.e., in terms of

$$\epsilon = (\omega\nu/U_\infty^2)^{1/6} \tag{2}$$

Allowing $\epsilon \to 0$ in the nondimensionalized, incompressible, Navier-Stokes equations while assuming that x is order one, one obtains the unsteady boundary layer equation to lowest order of approximation. The linearized unsteady boundary layer equation has been extensively studied in the literature (Moore[7], Lighthill[8], Lam and Rott[9], Ackerberg and Phillips[10]). At small distances from the leading edge, the unsteady boundary layer is quasi-steady and grows at the same rate as the steady boundary layer. At large distances from the leading edge, the unsteady boundary layer is controlled by the frequency and, to lowest order of approximation, behaves somewhat like a Stokes layer whose thickness remains constant, independent of x. The Stokes layer-like solution is independent of the upstream conditions and of the mean boundary layer. This type of asymptotic behavior occurs because the unsteady boundary layer equations are invariant under the Galilean transform

$$\left.\begin{aligned} \tilde{x} &= x - \int u^*(t)\,dt \\ \tilde{t} &= t \\ \tilde{u} &= u - u^*(t) \\ \tilde{v} &= v \end{aligned}\right\} \tag{3}$$

into an accelerated reference frame. Here, u and v are the streamwise and transverse velocity components in the boundary layer.

Then, when u_1 becomes constant far downstream we can take $u^* = \mathrm{Re}\, u_1 e^{-i\omega t}$ in this region and transform the fluctuating stream problem in the problem of an oscillating wall. But, in this region, the steady boundary layer is thick relative to the Stokes layer penetration distance $(\nu/\omega)^{1/2}$ and we might expect the unsteady flow to be the same as that produced by an oscillating wall bounded by a fluid that is at rest at infinity - which is precisely the Stokes layer problem.

Ackerberg and Phillips[10] and Lam and Rott[9] point out that the Stokes-like solution is essentially incomplete because it is uniquely determined independently of the upstream conditions that must always be imposed when solving a parabolic partial differential equation. The remaining portion of the solution is represented mathematically by an infinite set of 'asymptotic eigensolutions' of the unsteady boundary layer equation. They were originally discovered by Lam and Rott[9]. In the downstream region the unsteady boundary layer solution, therefore, consists of a Stokes-like solution plus the asymptotic eigensolutions, whose undetermined constants are found from the upstream conditions, as was actually done numerically by

Ackerberg and Phillips. One can say then that the asymptotic eigensolutions describe the approach of the full unsteady boundary layer solution to the Stokes-type solution.

The asymptotic eigensolutions only exist for $x > O(1)$. They are physically and mathematically independent both of each other and of the Stokes-type solution. Their amplitudes are determined by the behavior of the full unsteady boundary layer solution in the region $0 < x < O(1)$. They decay exponentially as they propagate downstream. In fact, they behave like

$$e^{-\lambda x^{3/2} - i\omega t}$$

where λ is a complex constant with Re $\lambda > 0$, so that the 'wave length' of their oscillation decreases like $x^{-1/2}$. This occurs because the asymptotic eigensolutions can produce no pressure fluctuations and must, therefore, behave somewhat like convected disturbances propagating into a region of decreasing streamwise velocity. Since a convected disturbance is one with zero convective derivative, $(\partial/\partial t) + U(\partial/\partial x)$, where U is the mean velocity, its phase ϕ must be $\omega t - \int dx/U$) and its wavelength must, therefore, decrease in the streamwise direction if U does. Near the wall

$$U \sim \zeta \sim y/\sqrt{x}$$

so that

$$\phi - \omega t \sim x^{3/2}$$

Thus, the wavelength of this disturbance decreases like $x^{-1/2}$ because it must penetrate into a region where the mean velocity decreases like $x^{-1/2}$ and not produce any pressure fluctuations. The importance of explaining this wavelength reduction mechanism was emphasized by Reshotko[11].

The asymptotic eigensolutions oscillate with a wavelength that decreases with increasing x while the mean boundary layer thickness increases. The cross stream pressure fluctuations, which are neglected in the unsteady boundary layer approximation, must, therefore, eventually become important and the asymptotic eigensolutions, which are based on this approximation, must then become invalid.

Goldstein[12] showed that one can obtain a new solution which applies further downstream than the unsteady boundary layer solution, by considering the limiting form of the governing equation as $\epsilon \to 0$ with $x_1 \equiv \epsilon^2 x$ (rather than x) held fixed. This leads to a solution that applies when $x = O(\epsilon^{-2})$. It is essentially the classical large Reynolds number - small wave number approximation to the Tollmien-Schlichting wave solution of the Orr-Sommerfeld equation (Lin[13] Tollmien[14], etc.), appropriately corrected for slow variation in boundary layer thickness. Thus, it decays exponentially fast in the downstream direction when x_1 is relatively small and exhibits exponential growth when x_1 is sufficiently large.

Goldstein[12] shows that this solution matches onto one of the asymptotic eigensolutions in some overlap domain and is, therefore, the natural continuation of this solution into the downstream region. The other asymptotic eigensolutions match with Tollmien-Schlichting waves that continue to decay.

The remaining portion of the asymptotic unsteady boundary layer solution, that is, the Stokes-type solution, remains uniformly valid in the downstream region and is, therefore, completely decoupled from the Tollmien-Schlichting waves.

At large Reynolds numbers, the Tollmien-Schlichting wave solution of the Orr-Sommerfeld equation is basically inviscid except in a thin region near the wall and in a critical layer about the point where the inviscid equation becomes singular. It is well known that the critical and wall layers coincide near the lower branch of the neutral stability curve. But, there are two inviscid regions outside this wall layer - a main inviscid region where the unsteady velocity is quasi-steady, and an outer region where the unsteady effects are important, but where the mean flow is nearly uniform. This 3-level structure is somewhat similar to the triple deck structure found in steady boundary layers - but, the transverse scaling is quite different here. The complete structure of the unsteady boundary layer found in ref. (12) is summarized in figure 1.

As we already indicated, the asymptotic eigensolutions of the unsteady boundary layer equation and the Tollmien-Schlichting wave solutions of the Orr-Sommerfeld equation match in the overlap domain. There are infinitely many asymptotic eigensolutions and the characteristic equation which determines the eigenvalues of the Orr-Sommerfeld equation has one root for each.

The progressive reduction in wave length of the asymptotic eigensolutions is a sort of 'tuning' mechanism which allows free stream disturbances to couple with Tollmien-Schlichting waves even when their streamwise wavelengths are vastly different. The Orr-Sommerfeld region acts like a high gain linear amplifier tuned to a very specific wavelength.

C. Comparison with Experiment

We now turn to the experiment of Shapiro[6] that we alluded to above. His unsteady disturbance was produced by an upstream acoustic speaker that generated a nearly plane acoustic wave, which propagated downstream parallel to the mean flow. The unsteady flow was, therefore, relatively two dimensional. The ratio of the acoustic wave length to the Tollmien-Schlichting wave length was about 30 in this experiment - so the acoustic wave behaved pretty much like a uniform oscillation of the stream.

Shapiro's[6] plate was relatively thick. But, it does correspond to the model described above since its noise radius was of the order of U_∞/ω.

Shapiro took his data with a narrow band filter and measured transverse velocity profiles of the streamwise velocity fluctuation in the boundary layer. His measured profiles were in close agreement with the theoretical Tollmien-Schlichting wave profiles near the upper branch of the neutral stability curve where the instability wave would have presumably grown well beyond the level of the Stokes shear wave solution (whose amplitude does not change with streamwise distance). Near the lower branch of the neutral stability curve, where the Tollmien-Schlichting wave is just beginning to grow, the measured profiles appeared to be a composite of the Stokes shear wave and a Tollmien-Schlichting wave. Moreover, the measurements near the lower branch show that the mean amplitude of the unsteady disturbance remains relatively constant with streamwise distance when averaged over a wave length but, the amplitude itself oscillates about this mean with a wave length that is roughly equal to the Tollmien-Schlichting wave length. The data is shown in figure 2. Thomas and Lekoudas[15] and Murdock[16] show that this behavior is precisely what one would expect if the solution consisted of Stokes shear wave plus a relatively small amplitude Tollmien-Schlichting wave. As we already indicated, Goldstein's asymptotic solution is of this form in the vicinity of the neutral stability curve.

Perhaps, most importantly, Shapiro's data show that the amplitude of the unsteady motion at any given point in the boundary layer increases linearly with the amplitude of the imposed free stream disturbance - indicating that the Tollmien-Schlichting waves are indeed generated by the imposed disturbance through a mechanism that is entirely linear in the unsteady motion.

II. GENERATION OF KELVIN-HELMHOLTZ INSTABILITIES AT A LEADING EDGE

A. General Background

It is well known that when small amplitude periodic flow occurs in the vicinity of a sharp trailing edge embedded in an otherwise steady flow, the pressure singularity that would otherwise occur in the infinite Reynolds number limit can often be relieved by the continuous shedding of vorticity downstream of the edge. One then says that a 'Kutta' condition is satisfied at the edge. Crighton[17] has shown that there are certain periodic trailing edge flows where the vortex shedding is represented mathematically by spatially growing instability waves of the Kelvin-Helmholtz type.

Suppose that an infinitely thin flat plate is embedded in a uniform inviscid flow on which a small amplitude unsteady motion is imposed. Unless the unsteady motion is a plane wave aligned with the plate, it will produce a square root singularity in the pressure at the leading edge. Now we have seen that the viscous motion near the edge is governed by the unsteady boundary layer equation which allows no transverse pressure variations. The viscous effects by themselves cannot, therefore, eliminate the pressure singularity in the inviscid solution. When there is no flow separation, the singularity does not appear in a real flow simply because all real plates have finite 'noise radii'. The fluctuations in angle of attack produced by the unsteady flow must be small enough so that the laminar boundary layer on the rounded nose does not separate. Tollmien-Schlichting waves only make thei appearance far downstream in the flow and can produce no upstream influence that can affect the pressure at the leading edge to say nothing of eliminating the pressure singularity that would occur at an infinitely sharp edge.

But, if the plate were embedded in a transversely sheared mean flow[18] with an inflectional velocity profile, as shown in figure 3, the incident disturbance could trigger a Kelvin-Helmholtz instability at the edge which could then eliminate or relieve the pressure singularity that would otherwise occur at that edge.

Such an instability wave is clearly detectable in figure 4, which is comprised of photographs of the flow over a wedge placed in a rectangular laminar jet. (The flow here is from left to right.) The photographs were taken during an edge tone experiment and the unsteady motion that triggered the instability wave could have been an acoustic wave reflected from the nozzle lip or a harmonic disturbance convected downstream by the mean flow, or perhaps both.

B. Theoretical Analysis

Suppose that the flow is inviscid. Since a transversely sheared mean flow is an exact solution of the inviscid equations of motion[18], it makes sense to calculate the small amplitude unsteady flow by linearizing these equations about a transversely sheared mean flow. As in the case of a completely uniform mean flow, the resulting unsteady motion will, in general, possess a square root singularity at the sharp leading edge unless the imposed unsteady motion is a plane wave aligned with the plate. Moreover, as long as we are willing to allow such a singularity, we can always require that the solution remain finite (i.e., that it does not 'blow up') at large distances from the edge.

However, it was shown by Goldstein[19] that this problem possesses an eigensolution, which involves a Kelvin-Helmholtz instability wave propagating downstream from the edge (and is consequently unbounded at infinity). This eigensolution also possesses a square root singularity at the leading edge. Then, since one can always add an arbitrary multiple of an eigensolution of a given problem to any particular solution of that problem and still satisfy the imposed boundary conditions, we can add this eigensolution to the particular solution that is bounded at infinity and adjust the arbitrary constant to cancel out the singularity at the leading edge.

The time periodic solution to a problem can be obtained by finding the long time (i.e., steady state) behavior of the solution to an initial value problem. A solution to such a problem that is identically zero before the incident disturbance is 'switched on' is said to be causal. The causal solution to the present problem is singular at the leading edge and involves a Kelvin-Helmholtz instability on the downstream flow (so that it is unbounded at infinity). Consequently, neither the solution that is bounded at infinity nor the solution that satisfies the leading edge 'Kutta' condition is causal. However, it is not at all clear that the steady state solution should be causal (Rienstra[20]). But, neither is it clear that the solution should be finite at infinity since the linearization is, at best, only valid in a local region near the edge and one cannot, therefore, impose a condition on the solution at large distances from that edge. Thus, at this point, it is not possible to establish which, if any, of these three solutions is correct.

Linearized inviscid theory of the type described above can be used to represent high Reynolds number turbulent flows when the turbulence intensity is sufficiently small and the unsteady interaction being calculated is completed in a time that is short relative to the decay (or turnover) time of the turbulent eddies (Hunt[21]). This linear theory of turbulence is usually referred to as 'rapid distortion theory'.

Then, in particular, we can use the inviscid flow model described above to represent the turbulent flow over a large flat plate placed downstream of the potential core in a turbulent airjet in the manner indicated in figure 5.

The assumptions of rapid distortion theory (perhaps more appropriately called rapid interaction theory in this case) are rather well satisfied in this flow. However, we must now use the 'gust' or 'hydrodynamic' solution of the inviscid equations (Goldstein[22,23]) to represent the incident turbulence. This solution is defined over the entire flow field even in the absence of the plate and (when the near flow is subsonic) it decays exponentially fast at infinity. It, therefore, has no radiation field (i.e., it is non-acoustic) and can be used to represent the turbulence that would exist in the absence of the plate. It has sufficient generality (i.e., it involves two arbitrary functions that can be specified as boundary conditions in any given problem) to represent an arbitrary incident turbulence field.

Since this solution exists independently of the plate, it will not, in general, satisfy the physically required boundary condition that its normal velocity component vanish at the surface of the plate. We must, therefore, add to it another solution that exactly cancels this normal velocity component at the plate (this is permissible since we are dealing with linear theory and superposition holds). However, the resulting solution will no longer exhibit exponential decay at infinity, but rather behave like an outgoing acoustic wave there. Thus, the plate 'scatters' the non-propagating motion associated with the gust or hydrodynamic solution into a propagating acoustic wave.

Much more interesting, however, is the fact that in both the causal and Kutta condition solutions the incident turbulence generates downstream propagating instability waves which, in the real flow, roll up and break to form new turbulence.

C. Comparison with Experiment

Goldstein[23] compared this analysis with Olsen's[24] measurements of the acoustic field produced by a large flat plate placed in the mixing region of a turbulent jet in the manner indicated in figure 5. His solutions satisfy causality.

Figure 6 is a comparison of Olsen's measurements of the sound radiated in the plane perpendicular to the plane of the plate in one third octave frequency bands as a function of angle measured from the nozzle inlet.

The upper part of the figure corresponds to the high frequency limit of the solution. Here, the instability waves are 'cut off' and the issues of causality and leading edge 'Kutta' condition are irrelevant.

The lower part of the figure corresponds to the low frequency limit. Here the instability waves have a large effect on the radiation field but, unfortunately, both the causal and Kutta condition solution lead to the same result. However, it is worth noting that they both differ significantly from the low frequency limit of the solution that is bounded at infinity and the agreement between experiment and theory would have been quite poor if the latter solution had been used.

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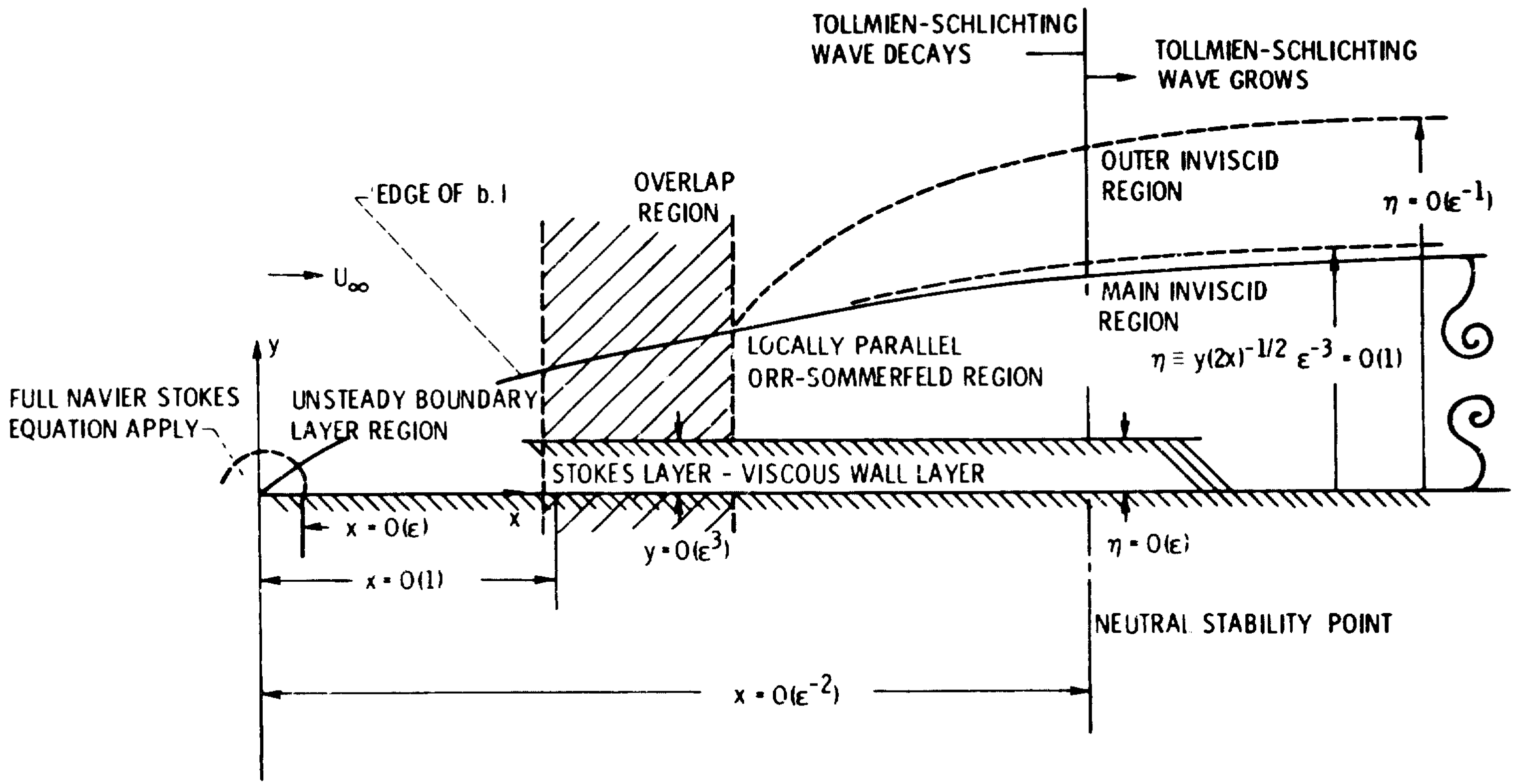

Figure 1. - Asymptotic structure of unsteady boundary layer; $\epsilon = (\nu\omega/U_\infty^2)^{1/6}$.

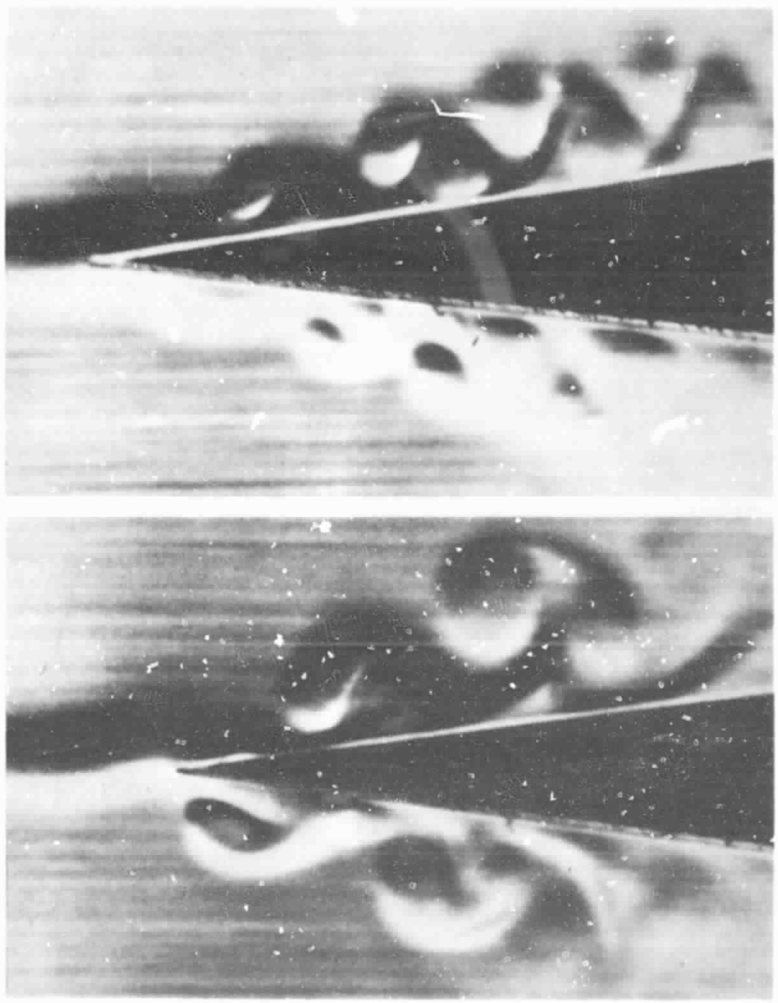

Figure 4. - Vortex shedding downstream of a leading edge.

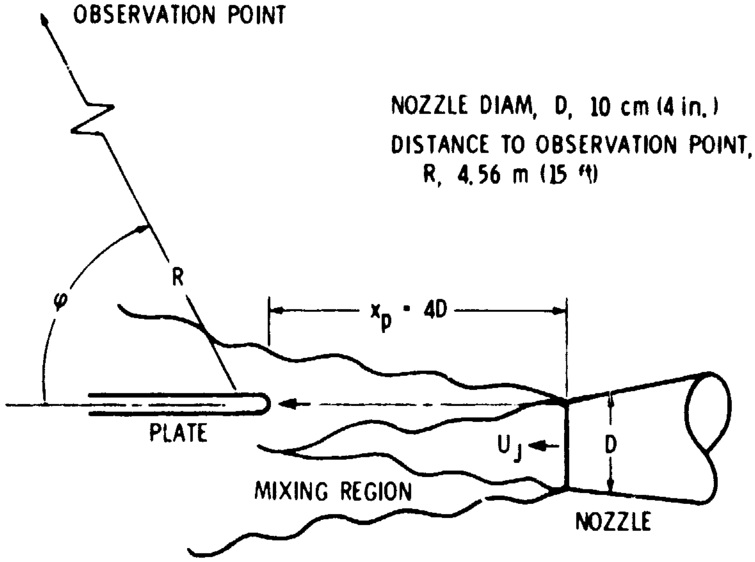

Figure 5. - Configuration of plate experiment

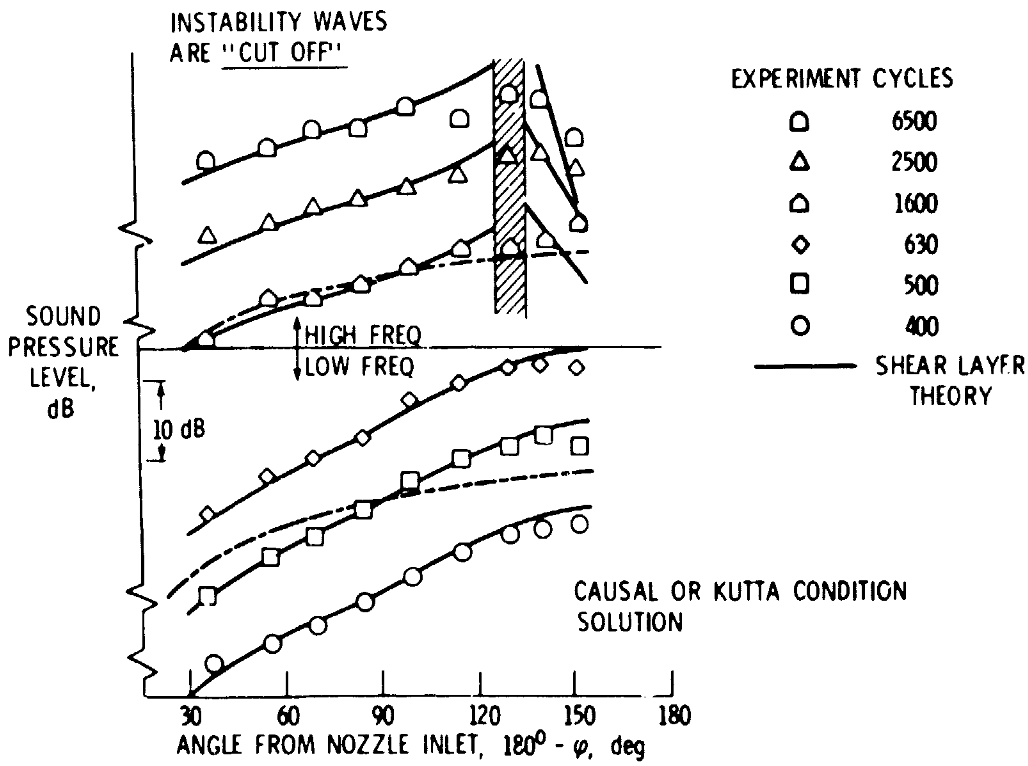

Figure 6. - Goldstein (J. F. M. 1979) comparison of causal or L. E. Kutta condition solution with data of Olsen for U_j = 700 l/S.